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# MAGNETO-OPTICS OF A THIN FILM LAYER WITH HELICAL STRUCTURE AND ENORMOUS ANISOTROPY

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The influence of an external magnetic field onto the magneto-optical properties of media with a helical periodical structure is discussed. The case of light normal incidence is considered, and it is assumed that the external magnetic field is directed along medium axis. The transmission and reflection of the light incident normal onto a thin film having a helical structure and being in external magnetic field is discussed. The Jones matrices for this system have been constructed. It is shown that at certain values of magneto-optical activity and local anisotropy a new diffraction reflection region rises. This system can be used as a narrow-band filter with controlled bandwidth. It is shown that at certain conditions this system can work also as an ideal optical diode or one-sided reflector. The specific features of eigenpolarizations are also discussed, and it is shown that the very dependences of eigenpolarizations onto external magnetic field direction conditions great values of nonreciprocal transmission.

Keywords: axial propagation; helical periodical media; large anisotropy; magneto-optical activity; optical diode; narrow-band filter; diffraction; anomalous transmission

#### INTRODUCTION

Influence of an external magnetic field on the optical properties of helical periodical media (cholesteric liquid crystals, chiral smectics, chiral ferronematics) is considered extensively both theoretically and experimentally in past research [1–4; and some papers that are cited therein]. First the changes of structure of medium caused by the reorientation of the director were taken into account [1–4]. However, there can be situations

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when the magnetic field does not cause changes in a structure of helical periodical media (HPM). It does take place, for example, for nonmagnetic HPM ( $\mu=1$ ), and also for ones with a negative magnetic anisotropy, when the external magnetic field is oriented along helix axes. Irrespective of a change of a structure, the magnetic field reduces the changes in a local tensor of a HPM dielectric permeability and, in particular, it raises the effect of Faraday. The coexistence of optical activity, caused by twistness of a structure and magneto-optical activity, brings to nonreciprocity [5–8].

As it is known an optical nonreciprocal birefringence is due to the difference of the refractive indices for two opposite directions of propagation of the electromagnetic wave (noninvariance in case  $\vec{k} \to -\vec{k}$ . It is not concerned with the Maxwell's equations invariance with respect to the change  $t \to -t$ , and is sequent of the special structure of material equations. There are different mechanisms of wave nonreciprocal birefringence. In the first case nonreciprocal birefringence is observed in magnetoelectrical crystals and is due to the simultaneous presence of magnetoelectric effect and magnetooptical activity [9–14]. In the second case nonreciprocal birefringence is observed in natural or structural gyrotropic media in an external magnetic field and is due to the simultaneous presence of optical activity and magneto-optical activity [15–19]. The effects of nonreciprocal birefringence are considered extensively both theoretically and experimentally also in previous research [20–25; and some others sited therein] and the interest in these effects has greatly increased in recent times.

The magneto-optical properties of HPM at light normal incidence are discussed in Eritsyan [5,6] and Gevorgyan [7]. In Kienja and Semchenko [8] the special features of Faraday effect in HPM at oblique incidence of light with the help of dynamic theory of diffraction is investigated.

In this paper the results of an exact analytical solution of a boundary-value problem of normal propagation of light through a layer of a HPM with a finite thickness, which is in an external magnetic field directed along an axes of a medium, are represented.

At local weak anisotropy of a medium the effects of an nonreciprocal birefringence also are rather weak and therefore do not represent large practical interest despite their uniqueness. However, as it is shown in Gevorgyan [26], multiple reflections in the layer of medium with finite thickness can enhance the effects of nonreciprocal birefringence by several orders of magnitude. We can expect that another mechanism of enhancement of effects of nonreciprocal birefringence can be the light diffraction on the periodical structure of medium, because in this case there is an interference of many (infinite number) waves, too. Really, as it is shown below, near the diffraction reflection region the enhancement of the non-reciprocal birefringence effects take place, and at large values of a local anisotropy of the medium or at large values of the amplitude of an external

magnetic field the nonreciprocal transmission becomes significant. At certain values of magneto-optical activity parameter or local anisotropy of the medium, such a system can work as an optical diode which passes the light in one direction and does not pass the light in the opposite direction.

In connection with a possibility of creation of artificial mediums with a helical structure [27–30], including media with given parameters, and also ferromagnetic spiral structures imitating properties of cholesteric liquid crystals on super-high frequencies, the possibilities of application of systems with properties circumscribed in the given paper increase and so does the interest in their study. As it is known, many modern theories of optics of complicated structures are based on the model of representation of a medium as multilayer system. And the solution of these problems is reduced to an exact solution of a problem of passing of light through a single layer with finite thickness.

A large anisotropy can be observed, e.g., in the vicinity of individual absorption lines, if one of the dielectric-constant components is much greater than the other, or if these components have different signs. A large value of magneto-optical activity parameter can be observed in the vicinity of gyromagnetic resonance. As it is known, for real media in the case of large anisotropy it is necessary to take into account the difference of the permeability tensor from unity. Thereupon in this paper we consider the general case; the problem is analyzed taking into account the anisotropy of both the permittivity and the permeability. We take into account also the changes of the values of components of local tensors of dielectric and magnetic constants and helix pitch. However, we limit ourselves to the consideration of the case when the considered media are still helicoidal.

# DISPERSION EQUATION

In case of presence of an external magnetic field directed along axes of a medium, the dielectric and magnetic constants tensors have the forms

$$\hat{\varepsilon}(z) = \varepsilon_m \begin{pmatrix} 1 + \delta_{\varepsilon} \cos(2az) & \pm \delta_{\varepsilon} \sin(2az) \pm ig_e/\varepsilon_m & 0 \\ \pm \delta_{\varepsilon} \sin(2az) \mp ig_e/\varepsilon_m & 1 - \delta_{\varepsilon} \cos(2az) & 0 \\ 0 & 0 & 1 - \delta_{\varepsilon} \end{pmatrix}, \quad (1)$$

$$\hat{\mu}(z) = \mu_m \begin{pmatrix} 1 + \delta_{\mu} \cos(2az) & \pm \delta_{\mu} \sin(2az) \pm ig_m/\mu_m & 0 \\ \pm \delta_{\mu} \sin(2az) \mp ig_m/\mu_m & 1 - \delta_{\mu} \cos(2az) & 0 \\ 0 & 0 & 1 - \delta_{\mu} \end{pmatrix},$$

$$\hat{\mu}(z) = \mu_m \begin{pmatrix} 1 + \delta_{\mu} \cos(2az) & \pm \delta_{\mu} \sin(2az) \pm ig_m/\mu_m & 0\\ \pm \delta_{\mu} \sin(2az) \mp ig_m/\mu_m & 1 - \delta_{\mu} \cos(2az) & 0\\ 0 & 0 & 1 - \delta_{\mu} \end{pmatrix},$$
(2)

where  $\varepsilon_m = (\varepsilon_1 + \varepsilon_2)/2$ ,  $\delta_{\varepsilon} = (\varepsilon_1 - \varepsilon_2)/(\varepsilon_1 + \varepsilon_2)$ ,  $\varepsilon_{1,2} = \varepsilon_{01,2} + \Delta \varepsilon_{1,2}$ ,  $\varepsilon_{01,2}$  are the principal values of the local dielectric-constant tensor in the absence of the external magnetic field,  $\Delta \varepsilon_{1,2}$  are the excitations of principal values of the local dielectric-constant tensor in the presence of the external magnetic field [31–34],  $\mu_m = (\mu_1 + \mu_2)/2$ ,  $\delta_\mu = (\mu_1 - \mu_2)/(\mu_1 + \mu_2)$ ,  $\mu_{1,2} = 7\mu_{01,2} + \Delta\mu_{1,2}$ ,  $\mu_{01,2}$  are the principal values of the local magnetic permeability tensor in the absence of the external magnetic field,  $\Delta\mu_{1,2}$  are the excitations of principal values of the local magnetic permeability tensor in the presence of the external magnetic field [31–34],  $\vec{g}_e = \vec{g}_e(\vec{H}_{ext})$  and  $\vec{g}_m = \vec{g}_m(\vec{H}_{ext})$  are the vectors of gyroelectric and gyromagnetic magneto-optical activities [31–34],  $\alpha = 2\pi/\sigma$ ,  $\sigma = \sigma_0 + \Delta\sigma$ ,  $\sigma_0$  is the helix pitch in the absence of the external magnetic field, and  $\Delta\sigma$  is the excitations of helix pitch in the presence of the external magnetic field [1–4].

In accordance with the generalized principle of kinetic coefficients [35], we have

$$\varepsilon_{ij}(\vec{H}) = \varepsilon_{ji}(\vec{H}), \mu_{ij}(\vec{H}) = \mu_{ji}(\vec{H}). \tag{3}$$

At the absence of absorption we also have

$$\varepsilon_{ij} = \varepsilon_{ii}^*, \mu_{ij} = \mu_{ii}^*. \tag{4}$$

Taking into account Equations (3) and (4) and separating  $\varepsilon_{ij}$  and  $\mu_{ij}$  into their real and imaginary parts we will have

$$\varepsilon_{ij}'(\vec{H}) = \varepsilon_{ji}'(\vec{H}_{ext}) = \varepsilon_{ij}'(-\vec{H}_{ext}), \mu_{ij}'(\vec{H}_{ext}) = \mu_{ji}'(\vec{H}_{ext}) = \mu_{ij}'(-\vec{H}_{ext}), \quad (5)$$

$$\epsilon_{ij}''(\vec{H}_{ext}) = -\epsilon_{ji}''(\vec{H}_{ext}) 
= -\epsilon_{ij}''(-\vec{H}_{ext}), \mu_{ij}''(\vec{H}_{ext}) = -\mu_{ji}''(\vec{H}_{ext}) = -\mu_{ij}''(-\vec{H}_{ext}).$$
(6)

Therefore

$$\Delta \varepsilon_{ij} = \Delta \varepsilon_{ij}(\vec{H}_{ext}^2), \Delta \mu_{ij} = \Delta \mu_{ij}(\vec{H}_{ext}^2), \tag{7}$$

while

$$ec{g}_e = ec{g}_e(ec{H}_{ext})$$

and

$$\vec{g}_m = \vec{g}_m(\vec{H}_{ext}). \tag{8}$$

Substituting  $\hat{\epsilon}(z)$  and  $\hat{\mu}(z)$  in Maxwell's equations

$$rot\vec{H} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t},\tag{9}$$

$$rot\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t},\tag{10}$$

let us seek their solution for the case of light propagation along the medium axis in the form

$$\vec{E}(z,t) = \sum \left\{ E_j^+ \overrightarrow{n_+} \exp\left[i\frac{2\pi}{\lambda} \sqrt{\varepsilon_m} (\chi + b)z\right] + E_j^- \overrightarrow{n_-} \exp\left[i\frac{2\pi}{\lambda} \sqrt{\varepsilon_m} (-\chi + b)z\right] \right\} \exp(-iwt), \quad (11)$$

where  $\chi = \lambda/(\sigma\sqrt{\varepsilon_m\mu_m})$ ,  $\lambda$  is the light wavelength in vacuum,  $\vec{n}_{\pm}$  are the unit vectors of circular polarizations. Substituting Equation (11) in Equations (9) and (10) we obtain the following dispersion equation with respect to b:

$$b^4 - a_1 b^2 + a_2 b + a_3 = 0, (12)$$

where  $a_1 = -2(1 + \chi^2 + G_e G_m - \delta_{\varepsilon} \delta_{\mu}), \quad a_2 = -4\chi(G_e + G_m), \quad a_3 = -2\chi^2(1 + G_e G_m + \delta_{\varepsilon} \delta_{\mu}) + (1 - \delta_{\varepsilon}^2 - G_e^2)(1 - \delta_{\mu}^2 - G_m^2) + \chi^4, G_e = g_e/\varepsilon_m,$  and  $G_m = g_m/\mu_m$ .

Thus, unlike the case  $g_e = g_m = 0$ , at which dispersion equation is biquadratic, in this case it has a linear term. Its solution has the form

$$b_{1,2} = \sqrt{\frac{s}{2}} \pm \sqrt{-\frac{a_1}{2} - \frac{s}{2} - \frac{a_2}{2\sqrt{2s}}},$$

$$b_{3,4} = -\sqrt{\frac{s}{2}} \pm \sqrt{-\frac{a_1}{2} - \frac{s}{2} + \frac{a_2}{2\sqrt{2s}}},$$
(13)

where

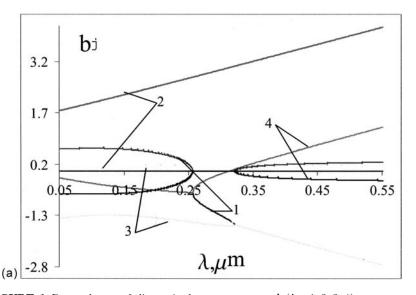
$$s = w - \frac{p}{3w} - \frac{a_1}{3}, w = \sqrt[3]{-\frac{q}{2} + v}, v = \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3},$$
$$p = -\frac{a_1}{12} - a_3, q = a_1 \left(a_3 - \frac{a_1^2}{36}\right) - \frac{a_2}{8}.$$

The problem considered here is solved for the general case  $\hat{\epsilon} \neq \hat{I}$  and  $\hat{\mu} \neq \hat{I}$ ,  $\hat{I}$  is the unit matrix, but here and further the concrete numerical calculations are fulfilled for the case  $\hat{\mu} = \hat{I}$ . Note only that the specific features of nonreciprocal birefringence observed here, such as the great values of nonreciprocal transmission, take place also in the case  $\hat{\mu} \neq \hat{I}$ .

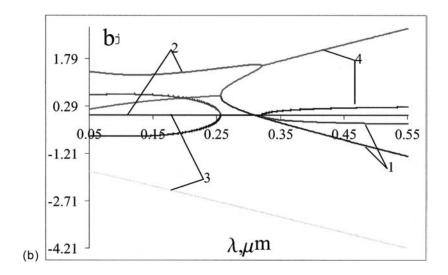
It is well known that in the absence of external magnetic field the lines  $b_j$  are symmetrical with respect to frequency axis (or  $\lambda$  axis) [2,4]. This symmetry disappears in the presence of external magnetic field: the curves of nonresonance  $b_j$  displace in one side parallel frequency axis, while the curves of resonance  $b_j$  displace in the opposite direction. These conditions stipulate wave nonreciprocal birefringence. As our calculations show,

the external magnetic field gives rise to a displacement of DRR. Unlike the case in which the displacement of DRR is caused by a change of a helix pitch, in this case it is always directed to the short-wave region (quadratic on  $g_e = g$  effect). At weak anisotropy this displacement is very small, and, for example, at  $g = \pm 0.1$  it makes only 1.5 A. The unique situation rises both at large values of a parameter of magneto-optical activity g or at large values of a local anisotropy  $\delta$ . Figure 1 shows the dependences of dimensionless wave vectors  $b_j$  onto wavelength  $\lambda$  at large values of  $\delta_\varepsilon = \delta$  and g. Figure 1 shows that

1. the external magnetic field brings to the rise of a new region (in long wavelength boundary of the spectrum), where two from four  $b_j$  become complex, as in DRR. Thus, a new DRR rises. This new DRR we shall call the second DRR, to distinguish if from the DRR, which exists at the absence of external magnetic field (or at small values of the amplitude of the external magnetic field), too. This DRR tends to infinite long waves. Let us note that the change of the direction of the external magnetic field to its inverse one practically does not influence on the frequency location of both DRR.



**FIGURE 1** Dependence of dimensionless wavevector  $b_j(j=1,2,3,4)$  onto wavelength  $\lambda$  at (a) g>0, (b) g<0, and (c) g=0. Solid lines correspond to  $Reb_j$  and dashed ones to  $Imb_j$ . Parameters are  $Re\varepsilon_m=0.25$ ,  $Re\delta=0.9$ ,  $Im\varepsilon_m=0$ ,  $Im\delta=0$ , and  $\sigma=0.4$  m $\mu$ . (See Color Plate I) (Continued).



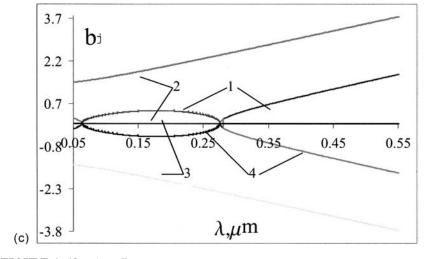


FIGURE 1 (Continued).

- 2. the external magnetic field gives rise to some displacement of the first DRR, which is very large now.
- 3. at an increase of the parameter g or  $\delta$ , the short-wave boundary of the second DRR is displaced to the long wavelength boundary of the first DRR and at certain values of these parameters all spectral region becomes diffractional.

Let us note that a new DRR also rises near to the line of absorption in the case when the optical activity of HPM molecules is taken into account [36].

# **BOUNDARY-VALUE PROBLEM FOR HALF-SPACE**

At first we consider reflection of light normally incident onto the planar half-space of HPM being in external magnetic field. The external magnetic field direction coincides with the helix axis direction, which is its turn is perpendicular to the boundary surface. Let us present the solution of this problem in the form

$$\vec{E}_r = \hat{R}\vec{E}_i,\tag{14}$$

where the indices i,r mark the incident and reflected waves, respectively,  $\hat{R}$  is the Jones matrix of the reflected field, and  $\vec{E}_{i,r} = E_{i,r}^+ \vec{n}_+ + E_{i,r}^- \vec{n}_- = \begin{pmatrix} E_{i,r}^+ \\ E_{i,r}^- \end{pmatrix}$  are the circular Jones vectors,

$$R_{11} = R_{22} = (\alpha_2 - \alpha_1 + \gamma_1 \beta_2 - \gamma_2 \beta_1)/\Delta,$$

$$R_{12} = (\alpha_1 \gamma_2 - \alpha_2 \gamma_1 + \beta_1 - \beta_2 + 2i(\gamma_1 - \gamma_2))/\Delta,$$

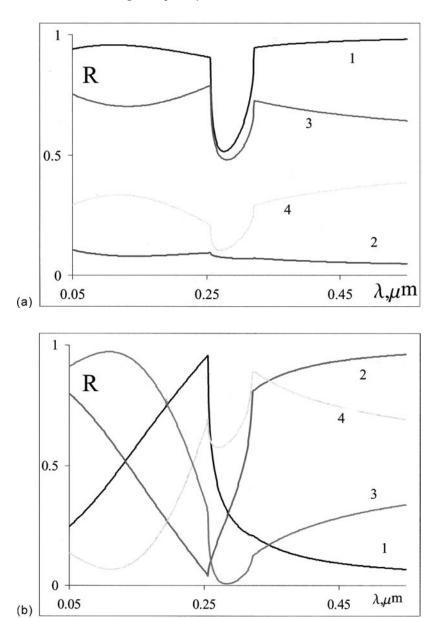
$$R_{21} = (\alpha_1 \gamma_2 - \alpha_2 \gamma_1 + \beta_1 - \beta_2 - 2i(\gamma_1 - \gamma_2))/\Delta,$$
(15)

where  $\Delta = (1 + \gamma_1)(\alpha_2 - \beta_2) - (1 + \gamma_2)(\alpha_1 - \beta_1)$ ,

$$\begin{split} \alpha_j &= i \frac{b_j [b_j^2 - \chi^2 - G_e G_m - (1 - \delta_\mu)(1 + \delta_\varepsilon)] + \chi [G_m (1 + \delta_\varepsilon) + G_e (1 + \delta_\mu)]}{\chi [b_j^2 - \chi^2 + G_e G_m + (1 - \delta_\mu)(1 - \delta_\varepsilon)] - b_j [G_m (1 - \delta_\varepsilon) + G_e (1 - \delta_\mu)]}, \\ \beta_j &= \alpha \frac{b_j [i G_m + (1 - \delta_\mu)\alpha_j] - \chi [\alpha_j G_m + i(1 - \delta_\mu)]}{[G_m^2 - (1 - \delta_\mu^2)]}, \\ \gamma_j &= \alpha \frac{b_j [i \alpha_j G_m - (1 + \delta_\mu)] + \chi [G_m + i(1 + \delta_\mu)\alpha_j]}{[G_m^2 - (1 - \delta^2)]}, \end{split}$$

 $\alpha = \sqrt{\epsilon_m/(\mu_m \epsilon)}$ ,  $\epsilon$  is the dielectric constant of the medium bordering with the half-space of HPM. We assume that the parameter of magneto-optical activity of the medium bounding with the half-space of the HPM is very small, so that we can neglect it.

Figure 2 shows the dependence of reflection coefficient R onto wavelength  $\lambda$  at g=0.275 (2a), at g=-0.275 (2b), and at g=0 (2c) for the incident light having the right (1) and the left (2) circular polarizations, and for linear polarizations along the direction of director on the entrance surface of HPM (3) and along the perpendicular direction (4). Throughout, we have assumed a right-handed HPM. Figure 2 shows that



**FIGURE 2** Dependence of reflection coefficient R onto wavelength  $\lambda$ . Other parameters are the same as in Figure 1. (See Color Plate II) (Continued).

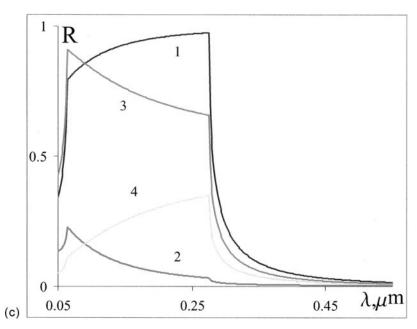


FIGURE 2 (Continued).

- 1. at given values of the problem there are two regions of diffraction reflection and transmission band between them. In these two regions the interaction between the light with both the right and left circular polarizations and the medium has a diffraction nature. The interaction between the light with two linear polarizations and the medium also has a diffraction nature. The high reflection in the transmission band has a Fresnel but not a diffraction character.
- 2. The change of the direction of the external magnetic field to the inverse  $(g \rightarrow -g)$  brings a large change in the character of the reflection of the light in the above-mentioned two DRRs and in the transmission band as well.

#### **BOUNDARY-VALUE PROBLEM FOR LAYER**

Now we consider the transmission and reflection of light in the case of normal incidence on the planar HPM layer, which is in an external magnetic field. The external magnetic field direction coincides with the helix axis direction, which is, in its turn, perpendicular to the boundaries surfaces. Let us present the solution of this problem in the form

$$\vec{E}_t = \hat{T}\vec{E}_i, \quad \vec{E}_r = \hat{R}\vec{E}_i,$$
 (16)

where the index t marks the field of the transmitted wave,  $\hat{T}$  is the Jones matrix of the transmitted field, and  $\vec{E}_t = E_t^+ \overrightarrow{n}_+ + E_t^- \overrightarrow{n}_- = \begin{pmatrix} E_t^+ \\ E_t^- \end{pmatrix}$  is the circular Jones vector of the transmitted wave. Solving the boundary-value problem for the elements of the Jones matrixes we obtain

$$R_{11} = \frac{1}{2\Delta} \sum e_{ijkl} [\gamma_{i}^{+} \alpha_{j}^{+} - \gamma_{i}^{-} \alpha_{j}^{-} + i(\gamma_{j} - \gamma_{i} + 2\alpha_{i}\beta_{j})] \gamma_{k}^{-} \alpha_{l}^{+} f_{k} f_{l},$$

$$R_{12} = \frac{1}{2\Delta} \sum e_{ijkl} [-\gamma_{i}^{+} \alpha_{j}^{+} - \gamma_{i}^{-} \alpha_{j}^{-} + i(\gamma_{i} - \gamma_{j} + 2\alpha_{i}\beta_{j})] \gamma_{k}^{-} \alpha_{l}^{+} f_{k} f_{l},$$

$$R_{21} = \frac{1}{2\Delta} \sum e_{ijkl} [-\gamma_{i}^{+} \alpha_{j}^{+} - \gamma_{i}^{-} \alpha_{j}^{-} - i(\gamma_{j} - \gamma_{i} + 2\alpha_{i}\beta_{j})] \gamma_{k}^{-} \alpha_{l}^{+} f_{k} f_{l},$$

$$R_{22} = \frac{1}{2\Delta} \sum e_{ijkl} [\gamma_{i}^{+} \alpha_{j}^{+} - \gamma_{i}^{-} \alpha_{j}^{-} - i(\gamma_{j} - \gamma_{i} + 2\alpha_{i}\beta_{j})] \gamma_{k}^{-} \alpha_{l}^{+} f_{k} f_{l},$$

$$T_{11} = \frac{1}{2\Delta} \sum e_{ijkl} (\alpha_{i}^{-} + i\gamma_{i}^{+}) [\alpha_{j}^{+} (\gamma_{k} - \gamma_{l}) + 2\alpha_{k}\beta_{l} \gamma_{j}^{-}] f_{j} f_{k} f_{l} \exp(-iad),$$

$$T_{12} = \frac{1}{2\Delta} \sum e_{ijkl} (\alpha_{i}^{-} - i\gamma_{i}^{+}) [\alpha_{j}^{+} (\gamma_{k} - \gamma_{l}) + 2\alpha_{k}\beta_{l} \gamma_{j}^{-}] f_{j} f_{k} f_{l} \exp(-iad),$$

$$T_{21} = \frac{1}{2\Delta} \sum e_{ijkl} (\alpha_{i}^{-} + i\gamma_{i}^{+}) [\alpha_{j}^{+} (\gamma_{k} - \gamma_{l}) - 2\alpha_{k}\beta_{l} \gamma_{j}^{-}] f_{j} f_{k} f_{l} \exp(iad),$$

$$T_{11} = \frac{1}{2\Delta} \sum e_{ijkl} (\alpha_{i}^{-} - i\gamma_{i}^{+}) [\alpha_{j}^{+} (\gamma_{k} - \gamma_{l}) - 2\alpha_{k}\beta_{l} \gamma_{j}^{-}] f_{j} f_{k} f_{l} \exp(iad),$$

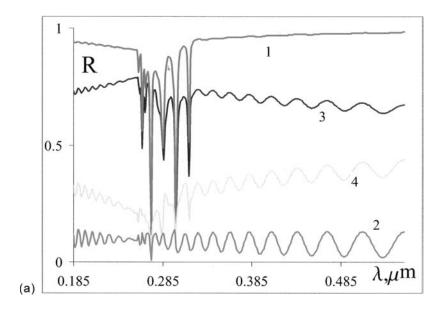
$$i, j, k, l = 1, 2, 3, 4,$$

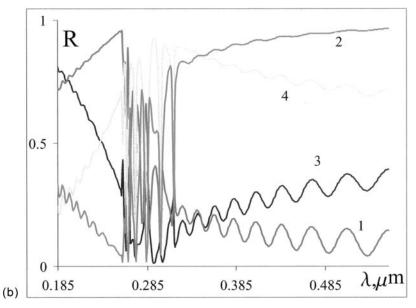
where

$$\begin{split} &\Delta = \sum e_{ijkl} \gamma_i^+ \alpha_j^- \gamma_k^- \alpha_l^+ f_k f_l, \quad \alpha_i^{\pm} = \alpha_i \pm \beta_i, \quad \gamma_i^{\pm} = 1 \pm \gamma_i, \\ &f_j = \exp(i2\pi \sqrt{\varepsilon_m \mu_m} b_j d/\lambda), \end{split}$$

 $\varepsilon$  is the dielectric constant of the medium bordering with the sample in both sides, and d is the layer thickness. Again, we assume that the parameters of magneto-optical activity of the media bordering with the sample in both sides are very small, thus we can neglect them.

Below the results of numerical calculations are presented. From Equations (16) and (17) one can compute the reflectance  $R = |E_r|^2/|E_i|^2$  and transmittance  $T = |E_t|^2/|E_i|^2$ , the absorption of the light in the layer bulk, the linear and the circular dichroisms. Throughout, we have assumed a right-handed HPM. For simplicity, we limit ourselves to consideration of the case  $n_0^2 = \varepsilon_m/\mu_m$ , but of course the presented theory is not limited to this case only. In other words, we considered the case of the minimum effect of the dielectric boundaries, where the diffraction, anisotropy, and non-reciprocal birefringence in the layer bulk will appear distinctively.





**FIGURE 3** Dependence of reflection coefficient R onto wavelength  $\lambda$ .  $d=10\sigma$  (a,b) and  $d=5\sigma$  (c). Other parameters are the same as in Figure 1. (See Color Plate III) (Continued).

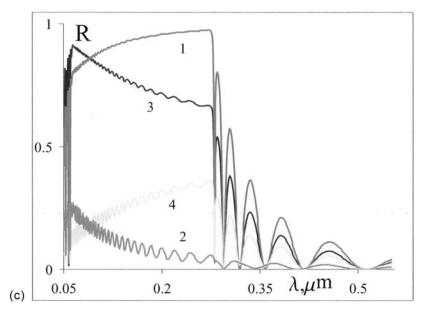
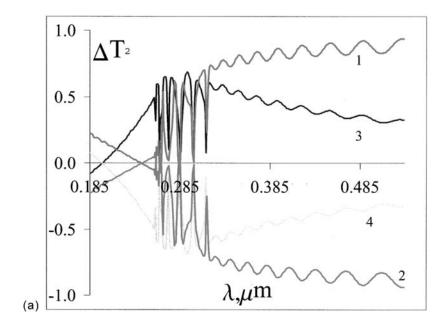


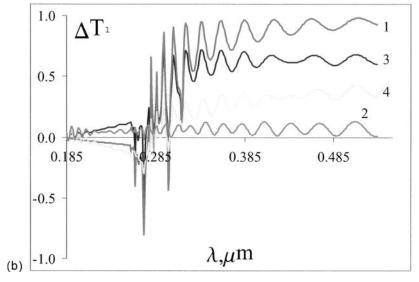
FIGURE 3 (Continued).

Figure 3 shows the dependence of the reflection coefficient R onto wavelength  $\lambda$ . The numbering and the parameters are the same as in Figure 2. As it is seen from Figures 2 and 3, the interaction between the light with both the right and left circular polarizations and the medium has a diffraction nature. A diffraction nature has also the interaction between the light with two linear polarizations and the medium. Oscillation behavior of reflection coefficient is caused as much by the diffraction of the light in a limited volume (but not by the diffraction of the light on the medium structure) as it is by Fresnel multireflections from layer boundaries as well (although we discuss the case  $n_0^2 = \varepsilon_m/\mu_m$ ).

Thus, the existence of a transmission band between diffraction reflection regions shows that this system acts as a narrow-band filter. As the width of the transmission band can be varied by changing the external magnetic field amplitude, we have a narrow-band filter with the controllable bandwidth  $\Delta \lambda$ . In the two DRRs this system acts as a filter (polarizer) for right- or lefthand elliptic polarizations, respectively, because it reflects the light with one-hand elliptic polarization and transmits the light with the opposite-hand elliptic one.

Figure 4a shows the dependence of the nonreciprocal transmission  $\Delta T_2 = T(g) - T(-g)$  onto wavelength  $\lambda$ . The numbering and the parameters are the same as in Figure 2. Here T(g) is the light transmission





**FIGURE 4** (a) Dependence of  $\Delta T_2$  and (b)  $\Delta T_1$  onto wavelength  $\lambda$ .  $d=10\sigma$ . Other parameters are the same as in Figure 1. (See Color Plate IV).

coefficient when the incident light direction coincides with the external magnetic field direction and T(-g) is the transmission coefficient when these directions are mutual reverse.

One can see that at certain values of parameters g or  $\delta$  the  $\Delta T_2$  are of the order of the unit. On the other hand, as  $\Delta T_2 = T(g) - T(-g)$ , it means that this system acts as an ideal optical diode, which completely passes light at its incidence on a system from the one side and does not pass it at all at its incidence on the system from the other side. Since in the absence of absorption  $|\Delta T_2| = |\Delta R_2|$  it means that this system can also act as one-sided reflector, reflecting fully the light at its incidence onto the system from the one side does not reflect it at all at its incidence from the other side. Note that at certain conditions some photonic crystals also act as an optical diode [37,38].

Figure 4b shows the dependence of the parameter  $\Delta T_1 = T(g) - T(0)$  onto wavelength  $\lambda$ . The numbering of curves are the same as in Figure 2. Here T(0) is the light transmission coefficient when the external magnetic field is absent.

One can see that at certain values of parameters g or  $\delta$  the  $\Delta T_1$  is of the order of unit. On the other hand, since  $\Delta T_1 = T(g) - T(0)$ , it means that this system acts as an optical lock. Of course, such systems can be used as one-sided reflectors as well.

Let's note that as the calculations show that at the absence of absorption the scattering matrix of this system is unitary, although it is not time-reversal, and low energy conservation takes place, too: T+R=1. At the presence of absorption we have T+R<1.

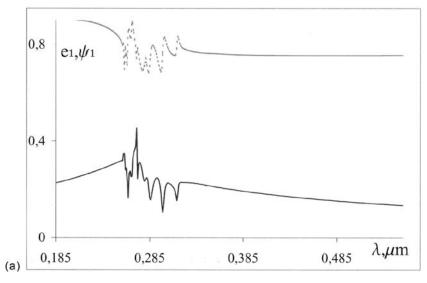
As the reflections between the light with two linear polarizations and with two circular ones and the medium has a diffraction nature, these polarizations are not EPs of the given system. As it is shown in [39,40], one can easily explain some special features of optical properties of optical systems by investigating the properties of their eigenpolarizations (EPs).

## SPECIAL FEATURES OF EPS

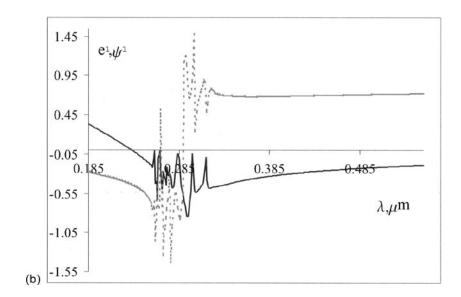
It is known that the EPs are the two polarizations that do not change after the propagation of light through an optical system. The properties of EPs of a layer with periodic helical structure were studied in [39,40], where it was shown that in the case of a weak local anisotropy of the refractive index they represent two near-circular polarizations (right- and lefthand). An important feature of periodical helical structures is that they show a complete selectivity not for circular polarizations but for EPs, the latter becoming circular only in the limit  $\delta \ll 1$ . The light with one of the EPs undergoes diffraction reflection, whereas the light with the other EP does

not undergo a diffraction reflection at all. Analysis of the influence of the local anisotropy  $\delta$  on the EPs of the HPM layer showed that their ellipticity decreases (in modulus) with increasing  $\delta$  (by modulus), and in the limiting case  $\delta \gg 1$ , they are transformed into orthogonal linear polarizations.

To reveal the characteristic properties of the EPs of a layer of HPM with large local anisotropy in an external magnetic field, we limit ourselves to consideration of the case  $n_0^2 = \varepsilon_m/\mu_m$ . Figure 5 shows the dependences of the ellipticity  $e_1$  (solid lines) and azimuth  $\psi_1$  (dashed lines) of the first  $\psi_2 = -\psi_1$ ) onto wavelength  $\lambda$  at g = 0.275 (5a), at EP  $(e_2 = -e_1,$ g = -0.275 (5b), and at g = 0 (5c). Local anisotropy  $\delta$  is of the order of unit ( $\delta = 0.9$ ). The figure shows that external magnetic field direction has strong influence onto EP response. In particular, when g > 0 the EPs represent two nearly-elliptic polarizations (right- and lefthand) and they undergo a comparatively weak change with the wavelength. Whereas in the case g < 0, although the EPs also represent two nearly-elliptic polarizations, they undergo a great change with the wavelength, changing their signs. But this means that, as in the case g > 0 the light with righthand elliptic polarization (with the first EP) undergoes a diffraction reflection and the light with the lefthand elliptic polarization (with the second EP) does not undergo a diffraction reflection at all, In the case g < 0 the light



**FIGURE 5** Dependence of ellipticity  $e_1$  and azimuth  $\psi_1$  of the first EP onto wavelength  $\lambda$ .  $d = 10\sigma$ . Other parameters are the same as in Figure 1. (See Color Plate V) (Continued).



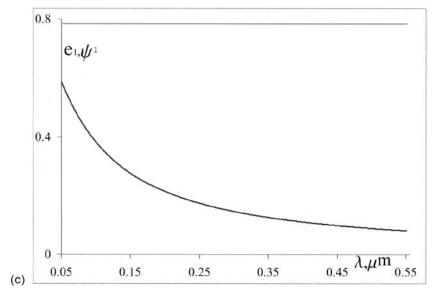


FIGURE 5 (Continued).

with righthand elliptic polarization undergoes diffraction reflection only in a certain wavelength region (in shortwavelength region). As for the rest of the wavelength region in the case g < 0, the diffraction reflection undergoes the light which has a lefthand elliptic polarization and vice versa. In short-wavelength region the light with the lefthand elliptic polarization does not undergo a diffraction reflection at all, whereas for the rest of the wavelength region the light with the righthand elliptic polarization does not undergo a diffraction reflection at all. In other words, as in the first DRR both at g > 0 and g < 0 where only the light with a righthand elliptic polarization diffracts, the nonreciprocal transmission is weak. In the second DRR frequency region at g > 0 only the light with a right-hand elliptic polarization diffracts, and at g < 0 only the light with a lefthand elliptic polarization is diffracted. Owing to this the nonreciprocal transmission is greater (of order of the unit) here. The particularities of the reflection of the light with various polarization are explained by these changes of EPs.

In conclusion let us note that the circumscribed specific features of HPM in an external magnetic field are due to the nonreciprocal reflection. This, in turn, means that a device which reflects nonpolarized light from its one side but not from its other side is impossible. The latter is due to the fact that the dielectric permittivity and magnetic permeability tensors satisfy the generalized principle of kinetic coefficients. Direct numerical calculations of reflection (transmission) coefficients at the nonpolarized light incidence from one and the other sides show the absence of any difference between them.

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